Sensitivity of Composite Indicators to Weights: Application to the EU Poverty Data

Tomáš Želinský[1] Francisco Azpitarte[2]

[1]Technical University of Košice & Charles University in Prague

[2]University of Melbourne & Brotherhood of St Laurence

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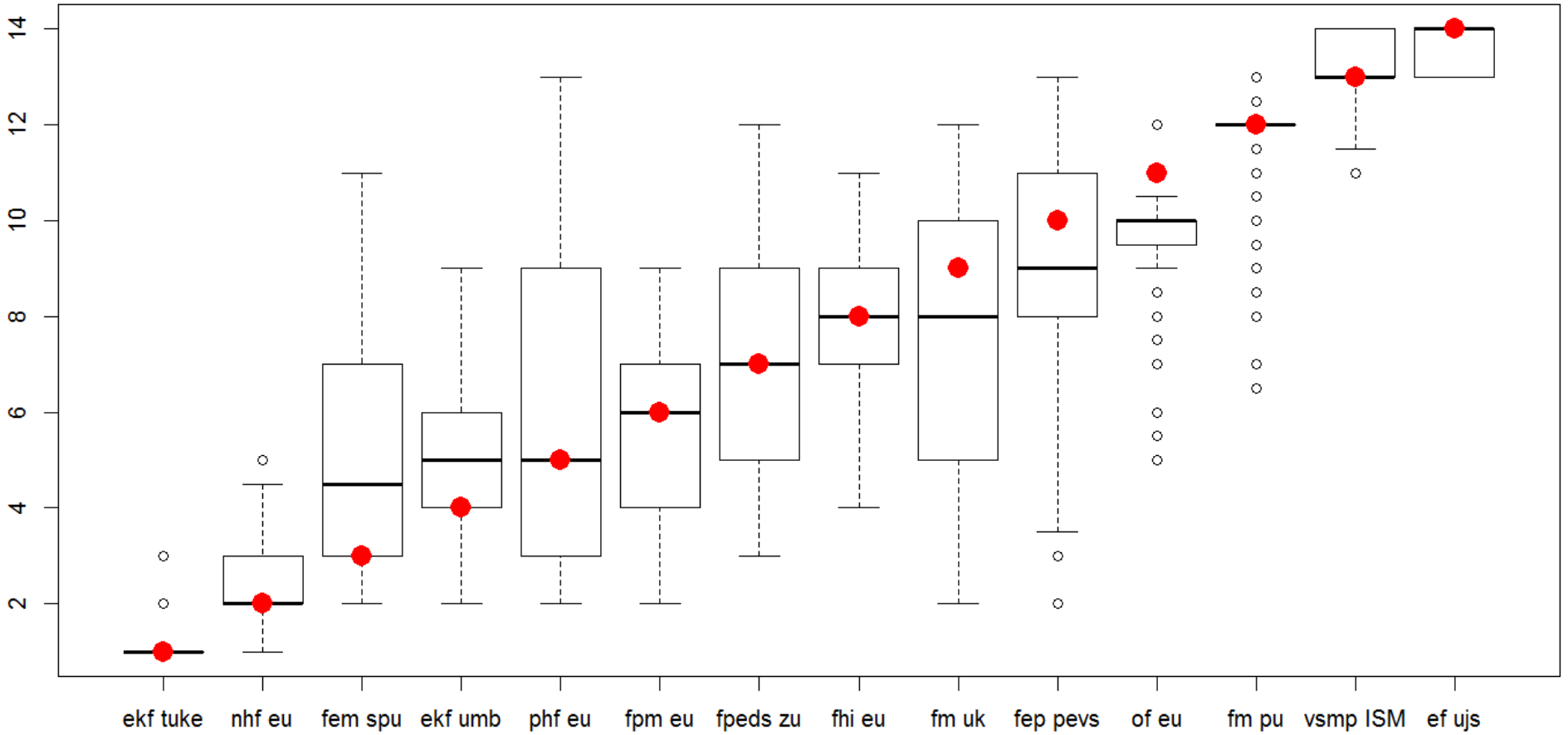
Motivation

- Ranking of the Slovak faculties (ARRA, Nov. 2015).
- The following groups of criteria:
  - education,
  - attractiveness of study (programmes),
  - research,
  - PhD study programmes,
  - grant successfulness.
- All groups are equally weighted.
- **Why?!** High level of grant successfulness & research activities $\rightarrow$ less time for teaching & teachers might slack off teaching!

Motivation

- OK, let's use weights.
- The setting:
  - Min weight: 5%, Max weight: 80%
  - All possible combinations (s.t. $\sum_{i=1}^{5} w_i = 100\%$) are taken into account (i.e. 3876 weighting vectors)
- *Do the ranks change significantly?*

Motivation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- How about something more serious?
- How about poverty?

- How about something more serious?
- **How about poverty?**

Questions

The measurement of poverty is complex: many methodological and normative choices

Especially true with multidimensional measures

1. Under which conditions can we claim *robustly* that poverty in region A is higher than in region B?
2. When can we say that poverty in a given region has *unambigously* declined or increased?

Introduction

- In the multidimensional poverty measurement: *weights* affect the identification and the depth of poverty
- How should be weigh the dimensions? No consensus
- Standard approach: equal weights and then robustness checks for a grid of vectors, e.g. official measure in the EU
- Not a good idea: poverty comparisons are in general extremely sensitive to weights
- Importance of dominance conditions

Measurement Framework

- Alkire and Foster (2011): very influential paper
- Counting approach to multidimensional poverty: valid for cardinal and ordinal dimensions
- Assume N units and D indicators of wellbeing. If $x_{nd} < z_d$ then n is deprived in d
- For each dimension $\exists\ w_d \in (0, 1)$ such that $\sum_{d=1}^{D} w_d = 1$
- The deprivation score for each individual is:

$$c_n \equiv \sum_{d=1}^{D} w_d \mathbb{I}(x_{nd} < z_d)$$

Necessary and Sufficient Condition

$M^A(k) \leq M^B(k)\ \forall k \in [0, v_2, \ldots, 1] \wedge \exists k | M^A(k) < M^B(k)$ for all possible weighting vectors $\mathbf{w}$ if and only if no sub-dimensional intersection headcount in A is higher than in B and there exists at least one sub-dimensional intersection headcount whose value is strictly lower in A relative to B.

- **People at risk of poverty or social exclusion:** one of the Europe 2020 Strategy headline indicators (to monitor progress towards the Europe 2020 strategy targets) adopted in 2010.
- Defined as the sum of persons who are:
  - at-risk-of-poverty and/or
  - severely materially deprived and/or
  - living in households with very low work intensity.
- In terms of A-F family of poverty measures, EU multidimensional poverty indicator has a form of headcount ratio $H(k, \mathbf{w})$ with $w_1 = w_2 = w_3 = \frac{1}{3}$ and $k = \frac{1}{3}$, i.e. the *union approach.*

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People at risk of poverty or social exclusion
1. At-risk-of-poverty

- **Data Sources:** EU-SILC (European Union Statistics on Income and Living Conditions)
  - Time period: 2008 - 2012
  - Spatial coverage: EU-28 plus: Iceland, Switzerland and Norway (Germany excluded)
  - Sample sizes (complete observations): almost 3mil. in total, ranging from 8,536 (IS-2009) to 52,433 (IT-2008)

- Living in households with equivalised disposable income **below 60 %** of the national equivalised median income (after social transfers).
- Modified OECD scale is applied (1 - 0.5 - 0.3)

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People at risk of poverty or social exclusion
2. Severe material deprivation

- Living in a household that cannot afford to pay for **at least four** out of nine items:
  1. to face unexpected expenses
  2. one week annual holiday away from home
  3. to pay for arrears (mortgage or rent, utility bills or hire purchase installments)
  4. a meal with meat, chicken or fish every second day
  5. to keep home adequately warm,

  or could not afford (even if wanted to):
  6. a washing machine
  7. a colour TV
  8. a telephone
  9. a personal car

People at risk of poverty or social exclusion

3. Very low work intensity

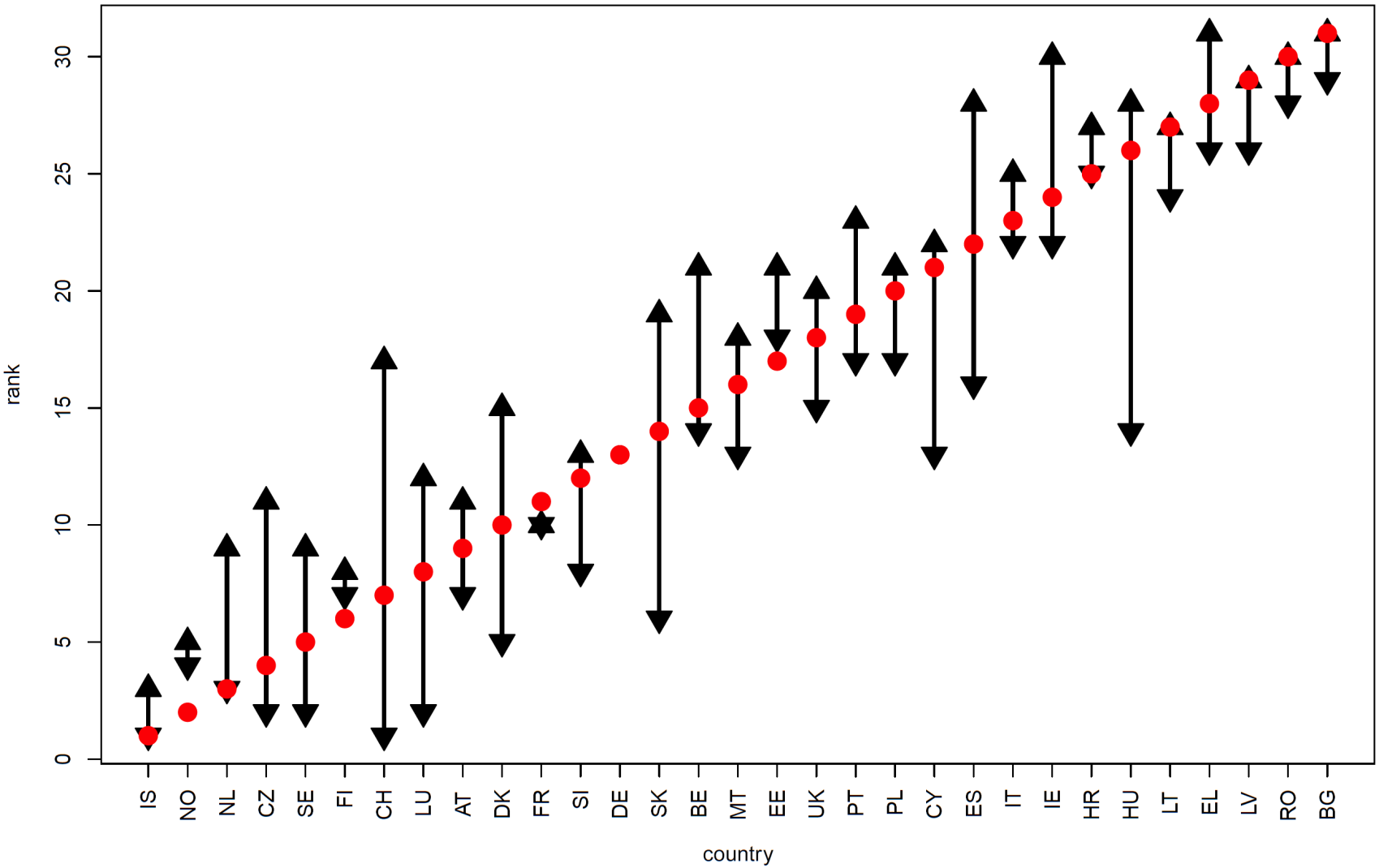
- Living in a household, where working-age adults (18-59) worked **less than 20 %** of their total work potential during the past year.
  - Based on the number of “months at work” and “workable months” of working age persons (18-64) in the household.

Do Weights Matter? #1

- In empirical literature sensitivity to weights checks are usually based on a very limited number of weighting schemes, e.g.:
  - Assigning a weight of e.g. 0.5 to one of the dimensions and 0.25 to each of the remaining two dimensions.
  - Assessment based e.g. on Spearman's ρ.
- Applying that approach to 2012 EU data we get the following results:
  - $(w_1 = 0.5, w_2 = 0.25, w_3 = 0.25) : \boldsymbol{\rho_s = 0.879}$
  - $(w_1 = 0.25, w_2 = 0.5, w_3 = 0.25) : \boldsymbol{\rho_s = 0.949}$
  - $(w_1 = 0.25, w_2 = 0.25, w_3 = 0.5) : \boldsymbol{\rho_s = 0.939}$

BUT:

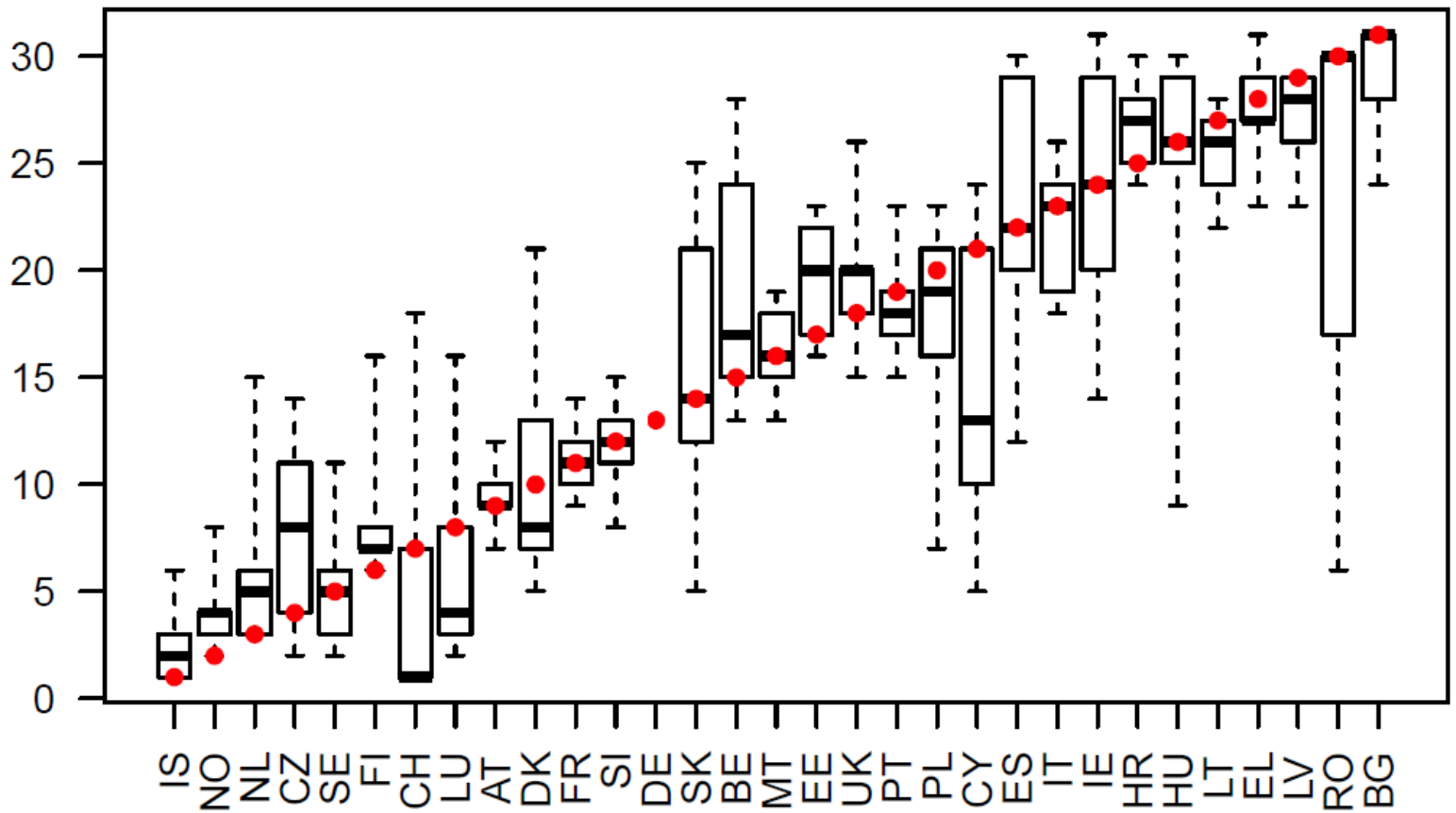
Ranks for the “common” weighting schemes



Do Weights Matter? #2

- Q: How do ranks change if we consider a wider range of combinations of k's and $\mathbf{w}$'s?
- Simple simulation:
  - $k = \left(\frac{1}{60}, \frac{2}{60}, \ldots, 1\right)_{N=60}$
  - Creation of weighting vectors $\mathbf{w}$ are based on *permutations with repetition* of elements of $\mathbf{w_0} = \left(\frac{1}{60}, \frac{2}{60}, \ldots, \frac{59}{60}\right)$ for which $\sum_{i=1}^{3} w_i = 1$, i.e. we have 1,605 weighting vectors.
  - For each combination of threshold k and weighting vector $\mathbf{w}$ ranks for all countries were computed.

Simulation of a wide range of k and $\mathbf{w}$ combinations



Empirical Application

Empirical strategy, two analyses:

1. Statistical testing of dominance conditions.
2. Finding maximum change in weights that preserves the initial ranks.

For any sub-dimensional intersection we run the test:

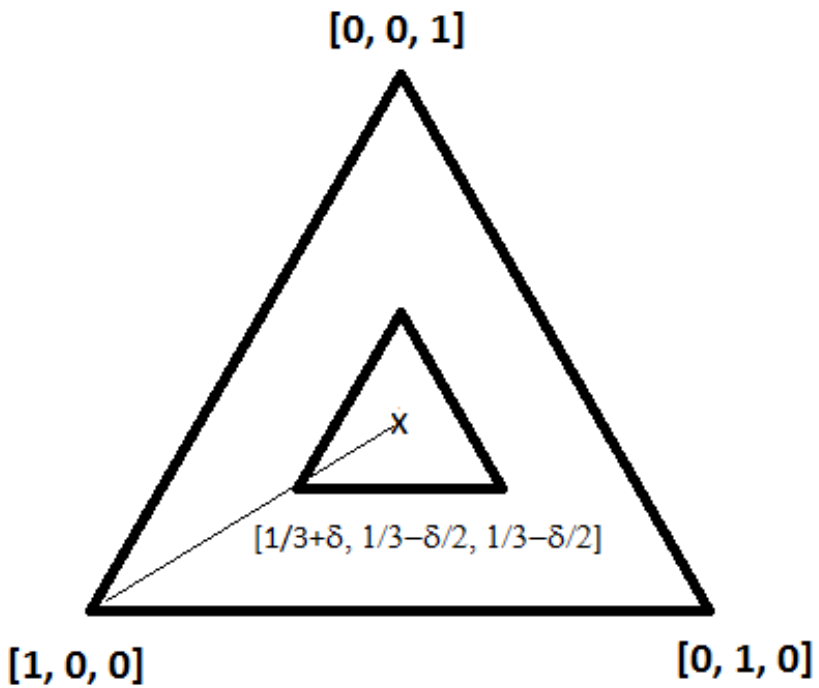
$$H_0 : \; M^A(k) = M^B(k)$$

$$H_A : \; M^A(k) > M^B(k)$$

$$T_{ij} = \frac{H_{ij}^A - H_{ij}^B}{\sqrt{\frac{\sigma^2_{H_{ij}^A}}{N^A} + \frac{\sigma^2_{H_{ij}^B}}{N^B}}}$$

where $\sigma^2_{H_{ij}^A} \equiv H_{ij}^A(1 - H_{ij}^A)$; $\sigma^2_{H_{ij}^B} \equiv H_{ij}^B(1 - H_{ij}^B)$

- **Q:** *How far can we go from equal weights while preserving the initial ranks in pair-wise comparisons?* (Permanyer, 2011)
- **The metric:** $\delta_{\text{max}} = \max\{\delta\}$ s.t. $\nexists$ reranking , $\delta_{\text{max}} \in \left[0, \frac{2}{3}\right)$



The following algorithm was used:

1. For each of the countries an initial ranking is detected for equal weights for each value of $k, k \in \{\frac{1}{240}, \frac{2}{240}, \ldots, 1\}_{N=240}$.
2. For each value of k the weights are redistributed: increasing the weight of one of the dimensions by δ and decreasing weights of the remaining dimensions by $\frac{\delta}{2}, \delta \in \{\frac{1}{120}, \frac{2}{120}, \ldots, \frac{2}{3}\}$, i.e. e.g.: $w_{1_i} = \frac{1}{3} + \delta_i, w_{2_i} = w_{3_i} = \frac{1}{3} - \frac{\delta_i}{2}$ for $i = 1, 2, \ldots, 80$.
3. For each pair of countries and each value of k a maximum value of δ which preserves the initial rankings (for that particular k) is identified.

Statistical evidence for dominance:

Year	Number of countries	Number of pairwise	Dominance [%] YES	Dominance [%] **NO**
2008	29	406	41.6	**58.4**
2009	29	406	42.5	**57.5**
2010	29	406	45.7	**54.3**
2011	30	435	54.6	**45.4**
2012	30	435	55.7	**44.3**

Results for 2012 data:

LT-LV	BE	BE		
IT	IT-UK	IT-UK		LT
PT-SE-SI	PT-SE-SI	PT-SE-SI	PT-SE-SI	PT-SE-SI
NL-NO-PL	NL-NO-PL	NL-NO-PL	NL-NO-PL	NL-NO-PL
IS-LU-MT	IS-LU-MT	IS-LU-MT	IS-LU-MT	IS-LU-MT
FI-FR-CH	FI-FR-CH	FI-FR-CH	FI-FR-CH	FI-FR-CH
SK-DK-EE	SK-DK-EE	SK-DK-EE	SK-DK-EE	SK-DK-EE
AT-CY-CZ	AT-CY-CZ	AT-CY-CZ	AT-CY-CZ	AT-CY-CZ
↓ countries above DOMINATE countries below ↓				
BG	**EL**	**HR**	**LT**	**LV**

IS	**LU**	**NL**	**NO**	**SE**
↓ countries above DOMINATE countries below ↓				
BE-BG-UK	BE-BG-UK	BE-BG-UK	BE-BG-UK	BE-BG-UK
EE-EL-ES	EE-EL-ES	EE-EL-ES	EE-EL-ES	EE-EL-ES
HR-IE-IT	HR-IE-IT	HR-IE-IT	HR-IE-IT	HR-IE-IT
HR-IE-IT	HR-IE-IT	HR-IE-IT	HR-IE-IT	HR-IE-IT
LT-LV-MT	LT-LV-MT	LT-LV-MT	LT-LV-MT	LT-LV-MT
PT-HU	PT	PT-HU	PT-HU	PT-HU
PL-RO-AT	PL-RO		PL-RO-AT	PL-RO-AT
CY-DK-FI	CY	DK-FI	CY-DK-FI	FR
FR-NL-SI			FR-NL-SI	
SK-CZ			SK	

Several dominance patterns can be identified. Based on the "longest path" we have e.g. the following clear dominance patterns:

$$\text{(IS-NO-SE)} \rightarrow \text{(AT)} \rightarrow \text{(FR)} \rightarrow \text{(MT)} \rightarrow \text{(EE)} \rightarrow \text{(LT)} \rightarrow \text{(LV)} \rightarrow \text{(BG)}$$

$$\text{(IS-NO)} \rightarrow \text{(NL)} \rightarrow \text{(FI)} \rightarrow \text{(MT)} \rightarrow \text{(EE)} \rightarrow \text{(LT)} \rightarrow \text{(LV)} \rightarrow \text{(BG)}$$

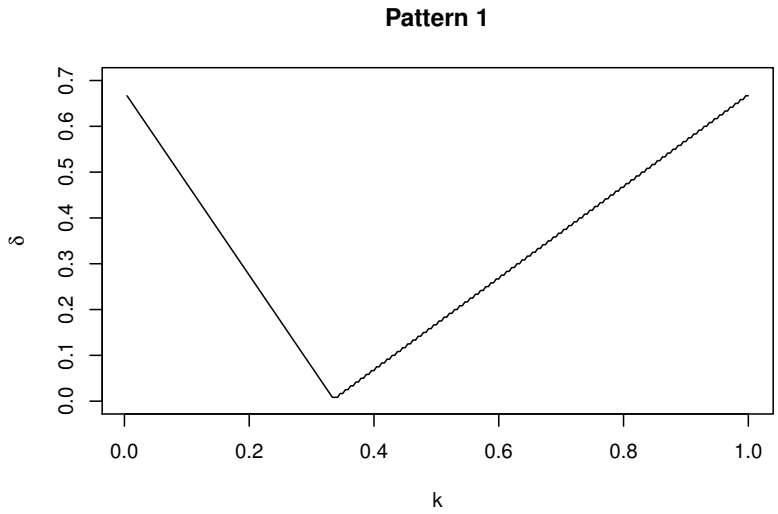
For pairs of countries where dominance can not be assumed (in terms of the official EU multidimensional poverty indicator), four main patterns of relationship between $\delta_{\max}$ and k have been identified, and they account for 94 % of all identified patterns:

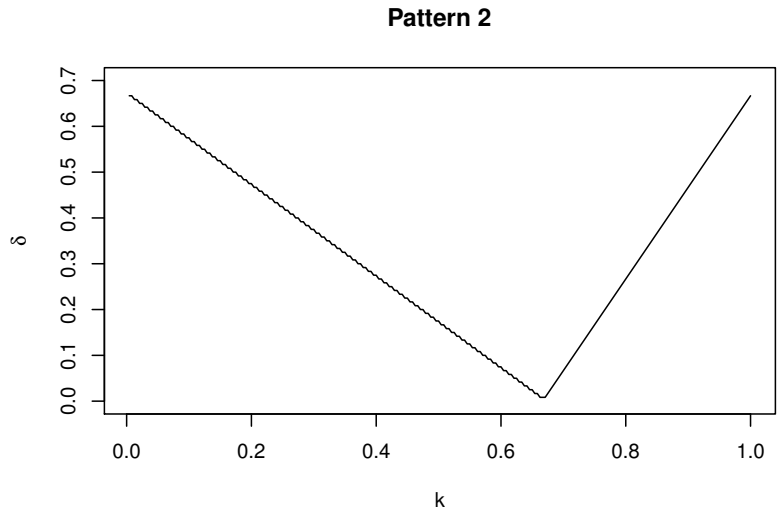
Pattern	2008	2009	2010	2011	2012
1	27,3	21,2	24,0	24,0	29,2
2	18,2	18,1	22,9	21,4	19,0
3	30,2	31,9	25,6	30,1	26,1
4	17,1	22,2	21,0	19,2	19,9
other	7,3	6,6	6,5	5,2	5,8

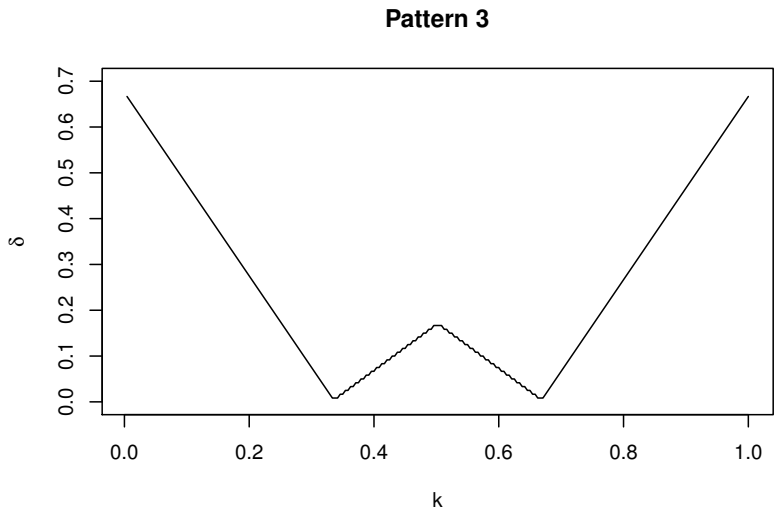
What are the patterns?

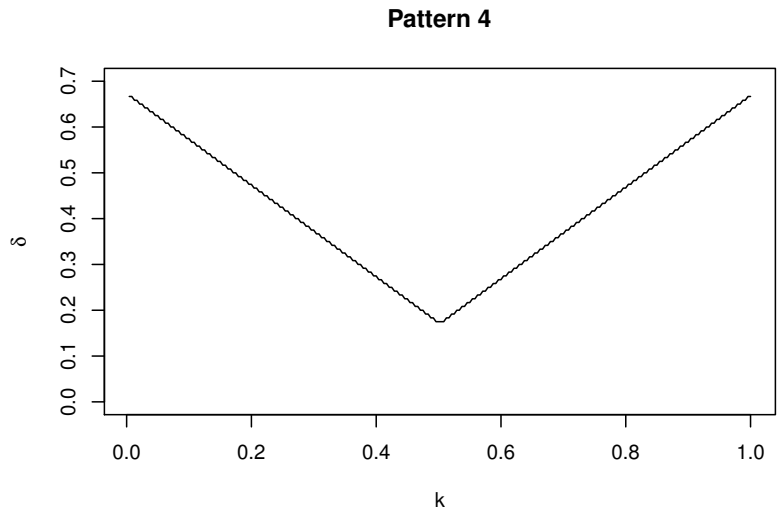
RESULTS: Cross-country Comparisons (EU)

2. δ's vs k's: patterns









Conclusions

- **Work in progress!**
- We derive dominance conditions to test the robustness of comparisons for the Alkire and Foster's family of poverty measures.
- Easy to apply. Cross-country and cross-years analyses: about 50 % of the comparisons are not robust.
- Poverty orderings are very sensitive to weights and cut-off values.
- Important for the analysis of time trends and cross-country comparisons: more attention should be given to sensitivity analyses.

Part of the research was financed by the **Go8 fellowship** and by the Slovak Scientific Grant Agency (grant **VEGA 2/0026/15**).

APPENDIX

$H_0: \ \Pr^A(S) = \Pr^B(S) \ \forall \ S \subset P_\rho(D)$
$H_a: \ \text{non } H_0$

$$ {}^iT = \frac{{}^iH^A - {}^iH^B}{\sqrt{\frac{{}^i\sigma^A_{M^A}}{N^A} + \frac{{}^i\sigma^B_{M^B}}{N^B}}} $$

where

- ${}^iH^A$ (or B) is the share of people satisfying the condition $i, i = 1, 2, \ldots, 15$ in country A or B
  - i.e. share of people deprived in d_1, d_2, d_3, $d_1\&d_2$, $\ldots$, $d_1\&d_2\&d_3$
- ${}^i\sigma^2_{M^A} \equiv {}^iH^A[1 - {}^iH^A)]; \ {}^i\sigma^2_{M^B} \equiv {}^iH^B[1 - {}^iH^B]$
- If we reject the null hypothesis for at least one i, we can conclude that A does not dominate B irrespective of weights.

Four possible outcomes:

- (1) H_0 is rejected for $H_A \leq H_B$ and H_0 is not rejected for $H_B \leq H_A$ or
- (2) H_0 is not rejected for $H_A \leq H_B$ and H_0 is rejected for $H_B \leq H_A$,
- (3) H_0 is rejected for both $H_A \leq H_B$ and $H_B \leq H_A$,
- (4) H_0 is not rejected for both $H_A \leq H_B$ and $H_B \leq H_A$.

	Dominance [%]	
Year	YES	NO
2008	41.6	58.4
2009	42.5	57.5
2010	45.7	54.3
2011	54.6	45.4
2012	55.7	44.3

"Perfect dominance": Ireland and Malta.

- But: increase in poverty over time.

				2011
			2010	2010
		2009	2009	2009
	2008	2008	2008	2008
2008	**2009**	**2010**	**2011**	**2012**
2012	2012	2012	2012	
2011	2011	2011		
2010	2010			
2009				

"Almost no dominance":

- Bulgaria: $H(2008) \leq H(2010)$ and $H(2009) \leq H(2011)$
- Switzerland $H(2009) \leq H(2008)$ and $H(2012) \leq H(2008)$